

Chain-wide and store-level analysis for cross-category management

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Abstract

When planning and implementing their price-promotions strategy, retail chain managers face the typical dilemma of “thinking globally, but acting locally.” In other words, they must plan their strategy, keeping in mind the global chain-level impact of their promotions, to deliver on the commitments made to manufacturers. At the same time, managers need to make sure that the implementation of such strategy takes into account the fact that each store caters to a different market with different needs and responses to marketing programs. Moreover, the retail chain manager must consider not only how the promotion of a brand affects competing brands and total category sales, but also how it could affect sales in other categories.

Our proposed model addresses these two important aspects of chain-wide and store-level cross-category analysis. First, our proposed factor-regression model takes store differences and longitudinal market shifts into account, thereby providing the retail chain manager with unbiased global, chain-level estimates. It also provides stable local estimates of cross-category promotion effects at the store level. Second, while allowing this flexibility, our proposed model is parsimonious enough over existing alternatives, making it particularly useful for chain-wide and store-level cross-category analysis.

We apply the proposed model to store-level data from one retail chain, comparing it with several competing approaches, and demonstrate that it provides the best balance between flexibility and parsimony. Most importantly, we show that the proposed model provides useful insights regarding cross-category effects at the chain-level, for individual stores, and their patterns across stores.

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Introduction

This study confronts two of the major limitations in traditional retailer pricing techniques, identified by Levy et al. (2004, p. xiv) in a recent editorial of the *Journal of Retailing*. The first problem involves setting the price for one product without taking into consideration its impact on other products. The second problem we address in our study is the system-wide character of retailers’ decision, where dif-

ferences in markets served by individual stores are often ignored.

Cross-category effects of price-promotions

Relatively few studies have focused on cross-category price-promotion effects, especially at a retail store level (Mulhern and Leone 1991; Walters 1991; Walters and MacKenzie 1988). As more and more firms leverage on their brand equity with brand extensions, one sees the prevalence of brands that transcend product categories such as shampoo and conditioner, paper tissue and napkins, toothbrushes and toothpaste, pasta and pasta sauce, among others. In some particular cases (e.g., Arm & Hammer), the brand associations are so

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robust that the brand is extended across a large number of unrelated product categories. Therefore, when evaluating the effects of sales promotions, brand managers must consider not only their impacts on competitors in the same product category, but also possible consequences to their “sister” brands in other categories.

Cross-category effects are particularly important in retail management for two main reasons. First, retailers’ main purpose of promoting a brand in a specific product category is not simply to sell more of the promoted brand, but also to increase sales of the product category and possibly generate more store traffic, resulting in higher sales in other product categories as well (e.g., Hruschka et al. 1999). A sales promotion that increases sales of the promoted brand, but not of the product category, is simply inducing brand switching, thereby cannibalizing regular sales of the competing brands. Second, most retailers now carry their own private label, a quintessential cross-category brand (Ailawadi and Harlam 2004). Thus, retailers are concerned about how their private label is affected by the promotions of national brands, and how their own promotion in one category affects sales across categories.

Thinking globally but acting locally

While managers of retail chains develop price-promotion policies that are consistent with the marketing strategy at the chain level, they should implement these policies in a way that is most effective at each store. On one hand, they want to have a chain-wide policy that reflects the chain’s pricing image and fulfills its trade-promotion agreements with manufacturers. On the other hand, they must deploy their price-promotions at the store level, taking into account how each particular local market responds to price discounts in each brand and product category. Therefore, it is critical to have access to reliable measures of promotion responses both at the chain level and the individual store (cf. Hoch et al. 1995).²

To obtain chain-level aggregate estimates, simply pooling data from all stores are likely to produce biased estimates, because they ignore the fact that these stores operate on diverse markets with distinct responses to the marketing mix, resulting in aggregation biases (cf. Blattberg and George 1991; Blattberg and Neslin 1990). These biases are likely to mislead the manager regarding the chain-wide impact of price-promotions. Thus, any investigation of promotional

effects must account for heterogeneity across stores. Estimates based on the data from each individual store, on the other hand, take into account the idiosyncrasies of the local market, but are often unreliable and incomplete due to limited observations. This is particularly problematic in a cross-category analysis, which involves multiple brands in each category, requiring a large number of cross-elasticities. More importantly, this approach also gives up potential benefits from the information obtained from other stores in the same chain.

One possible solution is to use a random-coefficients formulation, commonly applied in consumer choice modeling to account for unobserved heterogeneity (cf. Manchandra et al. 1999). This would produce unbiased estimates of the average, chain-level cross-brand and cross-category effects, as well as store-level estimates by taking advantage of the information available from all other stores. This “borrowing” of information is known to produce more reliable individual-level estimates (e.g., Blattberg and George 1991). Unfortunately, applying the usual random-coefficients approach to cross-category analysis would require a very large number of parameters to specify the multivariate distribution of the random-coefficients across stores, as we will explain in more detail later. This makes traditional random-coefficients models (either using a finite or continuous mixing distribution) often impractical for cross-category brand-level analysis as the number of brands and/or categories increases.

The main purpose of this paper is to investigate cross-brand and cross-category sales promotion effects both at the chain and store levels. Our intended contribution is two fold. First, we propose a new factor-regression model that offers a viable, parsimonious and relatively simple alternative to the random-coefficients models, which are widely used in the promotion response modeling (e.g., Hanssens et al. 2001). This proposed model makes it possible to account for cross-sectional and longitudinal variations in the regression coefficients, especially when the traditional random-coefficients-regression model is not feasible due to a very large number of coefficients. Second, we also attempt to provide store managers with more insightful summaries regarding the patterns of cross-brand and cross-category promotion effects across multiple stores. These cannot be fully obtained from a chain-level aggregate model or an individual store-level analysis. By providing a parsimonious way to account for variations in promotion cross-elasticities across multiple stores and over time, this study can improve store managers’ understanding of cross-category effects in category management (e.g., Levy et al. 2004).

It is important to note that these cross-category effects need to be measured at the brand level, rather than category aggregates for two main reasons. First, retailers and manufacturers can only implement category management by manipulating promotions at the SKU or, at the very least, brand level. Second, the aggregation of sales and, most impor-

² One might argue that a better assessment of cross-category promotion response might be obtained from the analysis of household-level basket data collected from loyalty programs. Obviously, this is only true for retailers that maintain such database at the customer level. Even then, basket analyses of loyal customer might not provide retailers with enough information to program their promotions in each store within their chain. Gupta et al. (1996) suggest that household panel data may not be representative of the population, resulting in incorrect price elasticity estimates. Moreover, to customize its strategy to the markets served by each store in the chain, the retailer will also need store-level cross-elasticity estimates (Hoch et al. 1995).

tantly, prices at the category level is likely to result in serious aggregation biases, misleading measures of price-promotion elasticities. Especially, since retailers tend to alternate promotions among brands over time, aggregate price indices at the category level are likely to lose valuable information about price variances and sales responses. By focusing on each brand, the framework we propose is able to uncover richer patterns of brand competition within and across categories. In addition, our empirical evidence also clearly exemplifies the limitation of the traditional random-coefficients approach; even for our relatively simple application with seventeen brands in two product categories, the traditional random-coefficients approach is not feasible, and a practical, simplified version of it does not perform as well as our proposed framework.

The next section briefly discusses relevant literature on store-level cross-category promotion effects. We then introduce our proposed factor-regression model, discuss estimation issues, and describe the data. Discussion of empirical results follows, including a predictive test. We conclude with managerial implications and future research directions.

Literature on store-level cross-category promotion response modeling

Compared to the growing literature on basket analysis using household-level scanner data, cross-category promotion effects at store level are relatively under-researched. Among the first to tackle this problem are Walters and MacKenzie (1988), Walters (1991), and Mulhern and Leone (1991), who develop store-level cross-category sales response models using regression methods. Walters and MacKenzie (1988) use data from two stores (for purposes of validation) from a large supermarket chain to examine the impact of price promotions on store traffic, sales of promoted and nonpromoted products, and store performance with a structural equation approach. In their study, all cross-category relationships are assumed to arise through store traffic and at the category level, rather than at the brand level, where managers are actually able to implement their promotion policies. Walters (1991) extends the Walters and MacKenzie's (1988) study by considering two stores from competing retailers. The study finds that the pricing and promotion of brands in one category affect sales of brands in a complementary category. He also finds that discounting a brand in one store decreases sales of the same brand in another store, and decreases sales of the competing brands in other stores. Mulhern and Leone (1991) examine promotion effects on store profitability in the presence of demand interrelationships, using scanner data from two stores. Their findings confirm those of Walters (1991) within a store.

These studies focused on a single store or a small set of competing stores (e.g., two stores), with an implicit assumption that promotion sensitivity and cross-category promotion effects would be generalizable across all stores. However, this

assumption ignores the possibility that each store in a retail chain serves a distinctive trade area responding differently to price promotions. Hoch et al. (1995) estimate price elasticities for each store in multiple categories, but for a larger sample of stores.³ However, they restrict all cross-elasticities to be the same for all stores (Hoch et al. 1995, p. 22), ignoring the fact that these stores operate in diverse markets and not accounting for any cross-category promotion effects.

In contrast to the studies reviewed above, our factor-regression model provides retail category managers with richer insights into the patterns of cross-brand and cross-category promotion effects without any restriction on the patterns. Our model also fully accounts for the fact that each store covers a distinctive market with different price sensitivity in the various brands and categories. Furthermore, we decompose the cross-category effects into chain-wide, store-specific, and time-specific components. In other words, the proposed factor-regression formulation produces average cross-brand/cross-category promotion elasticity estimates at the chain level, helping the chain manager to “think globally.” At the same time, the model also allows category managers to obtain individual estimates for each store, taking advantage of all the available data across stores, thereby providing them with valuable information to “act locally.” Moreover, the factor structure uncovered by the model helps managers understand how stores within the retail chain differ in their responsiveness to price promotions and in cross-category promotion effects, and how these differences might relate to demographic characteristics of the markets served by each store.

Factor-regression model

Consider the situation of a retail chain consisting of multiple stores $s = 1, 2, \dots, S$, trying to understand how weekly sales Y_{jst} of multiple brands $j = 1, 2, \dots, J$ collected across multiple categories over time ($t = 1, 2, \dots, T$) are affected by the individual net prices of all brands across all categories. Because different stores cater to different mixes of customers, the retailer would want to allow for heterogeneity in the response to price across stores. Due to seasonality and other possible time-dependent effects, the retailer would also want to account for nonstationarity in the parameters of the sales response model. This would require a system of J seemingly unrelated regressions estimated over time and across stores:⁴

$$Y_{jst} = \beta_{jst} X_{st} + \varepsilon_{jst} \quad (1)$$

³ There are several recent attempts to explain the variation in price elasticities across stores without explicitly modeling cross-category effect (e.g., Montgomery 1997; Mulhern et al. 1998). Karande and Kumar (1995) also investigate the variation in the promotion elasticities across brands by relating the elasticities estimates to brand characteristics.

⁴ Since the predictors are the same across all equations, Ordinary Least Squares is as efficient as the Seemingly Unrelated Regression estimator.

where X_{st} is a $(J+1)$ —dimensional vector containing the prices of all J brands (plus a column of 1 for the intercept), and β_{jst} is the vector of regression coefficients.

The model described above (Eq. (1)) is obviously unfeasible as it uses negative degrees of freedom. A common solution to this problem would be to specify the system of regressions in (1) as a random-coefficients model, assuming that the vector of regression coefficients β_{jst} stable over time – thereby ignoring nonstationarity – and has a multivariate normal distribution across stores, with a $J(J+1)$ square covariance matrix Σ_β . However, aside from assuming stationarity in the response parameters over time, this classic random-coefficient solution is rarely feasible for even a small problem with two categories. For example, consider a simple application with two product categories, each with 10 brands; the model specified in (1) would involve $20 \times 21 = 420$ random regression coefficients, and the full random-coefficients regression model would require the estimation of $420 \times 421/2 = 88,410$ covariance terms!

One way to make the random-coefficients regression feasible in these situations would be to assume independence of the promotion effects across brands and stores (an assumption that we will use in one of the models we use as a benchmark in our empirical tests later on). This assumption is obviously unrealistic, as it implies that preferences for one brand are independent from those for other brands in the same and other product categories.

In order to overcome these problems, we propose a random-coefficients formulation in which the regression coefficients are assumed normally distributed across stores and over time. However, instead of estimating all items in the covariance structure Σ_β , we specify a principal-components decomposition of this covariance of the random-coefficients, so that:

$$\beta_{jst} = \mu_j + \lambda_j V_s + \gamma_j W_t + \xi_{jst} \quad (2)$$

where $\mu_j = (J+1) \times 1$ vector of means for the random-coefficients distribution; $V_s = p \times 1$ vector of factor scores for store s , accounting for unobserved heterogeneity; $\lambda_j = (J+1) \times p$ vector of loadings for brand j on the heterogeneity factors; $W_t = q \times 1$ vector of factor scores for week t , accounting for nonstationarity; $\gamma_j = (J+1) \times q$ vector of loadings for brand j on the nonstationarity factors; $\xi_{jst} = \text{i.i.d.}$ random error with variance $\sigma_{j\xi}^2$.

With this formulation, we decompose the covariance of random coefficients into a set of p factors accounting for heterogeneity across stores and q factors accounting for shifts in the regression coefficients over time. This cross-sectional and temporal factor structure captures the inter-dependence among the brands and categories across stores and over time, while maintaining an appropriate balance between model fit and parsimony. In other words, while we avoid the unrealistic assumptions that all stores are alike and that brand preferences are independent within and across categories, we also keep our model feasible for the data.

Model estimation

Combining (1) and (2), the system of regressions can be re-written as,

$$Y_{jst} = \mu_j X_{st} + \lambda_j (V_s X_{st}) + \gamma_j (W_t X_{st}) + \xi_{jst} X_{st} + \varepsilon_{jst}. \quad (3)$$

If the factor scores V_s and W_t were known, estimates of the model parameters could be easily obtained through Feasible Generalized Least Squares. We propose a simple, easy-to-implement approach to estimate model parameters and factor scores, using simulated maximum-likelihood via E-M algorithm with standard error corrections. Details about this algorithm are omitted due to space constraints, but can be obtained directly from the authors.

Interpreting the results from the proposed model

The model described in (1)–(3) is a multivariate system of random-coefficients regressions, with mean coefficients μ_j across stores and over time, and a variance-components decomposition of the covariance of random coefficients,

$$\Sigma_\beta = \lambda' \lambda + \gamma' \gamma + \Sigma_\xi, \quad (4)$$

where Σ_ξ is a diagonal matrix of variances $\sigma_{j\xi}^2$.

The mean coefficients μ_j provide the chain manager with an assessment of the average chain-wide cross-elasticities for brand j , after accounting for the differences in response across stores and any fluctuations over time. Because the average elasticities are estimated after accounting for store heterogeneity and nonstationarity, these average estimates do not incur the aggregation biases from pooled regressions, providing the retail chain manager a clearer picture of system-wide promotion effects.

The proposed factor-regression model can also be viewed as a shrinkage-regression model where data from all stores are used to improve store-level estimates. Individual estimates for a store s can be directly obtained from the model, that is, $\beta_{js} = \mu_j + \lambda_j V_{js}$. Because these are shrinkage estimates, they take advantage of all the available information in the data (including from all other stores), thereby better reflecting how store s is likely to respond to price-promotions in the near future. We will later test this advantage in our empirical application.

In addition, the heterogeneity and nonstationarity factor structures provide useful graphical summaries of how stores differ in their price responses and how these responses shift over time. Note that the first term in the right-hand side of (4) represents the covariance in price response across stores accounted by the p heterogeneity factors, while the second term reflects the covariance in price response over time, captured by the q nonstationarity factors. For example, if the loadings (λ) of two random-coefficients point in the same direction of the latent (factor) space, these coefficients are

positively correlated across stores. Thus, stores with factor scores (V_s), located in the latent space pointed by the loadings, will have a higher-than-average response on both coefficients. This feature, which we will demonstrate later in our empirical application, allows the retail manager not only to measure the chain-wide effects, but also to obtain a graphical summary of how each store deviates from these chain-wide averages.

Similar insights can be drawn from the nonstationary scores (W_t) and loadings (γ); two random-coefficients with loadings pointing in the same direction in the latent nonstationary space are positively correlated over time, that is, have a similar time trend. For example, a unidimensional nonstationary solution ($q = 1$) would imply that all random-coefficients follow the same general trend line. A multidimensional nonstationary solution would allow more flexible time trends across response coefficients.

Empirical analysis

To investigate store-level cross-category promotion effects with our proposed model, we analyze weekly store-level data from the Dominick's chain, made available by the James M. Kilts Center, GSB, University of Chicago. These data consist of sales and prices for 9 brands of toothpaste and 8 brands of TOOTHBRUSH in 66 stores.⁵ We chose these two categories because of their close connection in terms of consumption, and because of the prevalence of cross-category brands. Out of 105 weeks, we use 78 weeks for model estimation, and hold out the remaining weeks for predictive tests.

We apply the model described in (1)–(3) to log-sales and log-prices, so that the price parameters are directly interpretable as (cross)-elasticities and the intercepts can be interpreted as the brand value after accounting for price (cf. Blattberg and Neslin 1990).⁶ In order to determine the numbers of heterogeneity (p) and nonstationary (q) factors we fitted the model for a range of values and chose the solution with the lowest Bayesian Information Criterion (BIC), arriving at a two-factor solution for both heterogeneity and nonstationarity, as shown in Table 1. The BIC change patterns are consistent and monotonic, implying the choice of an optimal number of factors is robust.

Before we interpret the results from our factor-regression model, we compare its goodness-of-fit and predictive performance with five competing models (i.e., an aggregate model, two models with store heterogeneity, and two with nonstationary assumptions): (a) an aggregate model estimated by pooling the data across all 66, (b) a store-level

Table 1
Model selection using the Bayesian Information Criterion

	Number of nonstationarity factors ^a				
	1	2	3	4	5
Number of heterogeneity factors ^a					
1	79784	79502	81438	84481	87622
2	77317	76856	78720	81501	83972
3	78661	78384	80184	82642	85560
4	80973	80900	82933	85258	87959
5	85258	84607	85462	88406	91747

Note: Boldface type indicates the selected model formulation.

^a BIC.

model fitted to each of individual store, (c) an independent random-coefficients model, assuming stationarity,⁷ (d) a nonstationary random-coefficients model, assuming homogenous stores (i.e., aggregate cross-elasticities vary over time), and (e) Kalman-filtering model. These comparisons highlight the importance of accounting for both store heterogeneity and nonstationarity using all the available data with a parsimonious formulation. Their goodness-of-fit and predictive tests on the 27 weeks of holdout data are shown in Table 2.

As one would expect, the aggregate model produces the worst and the store-level model produces the best goodness-of-fit. The former is too restrictive, while the later has more opportunities to adapt to the data, including random noise. Due to its independence assumption in the distribution of the coefficients, the independent stationary random-coefficients model produces worse fit than the proposed factor-regression model. In addition, as we can find from nonstationary random-coefficient model and Kalman-filter model, controlling for nonstationarity but not accounting for heterogeneity does not improve prediction in the hold-out sample. These predictive fit comparisons clearly show that our factor-regression model is parsimonious yet flexible enough to capture the patterns of heterogeneity and nonstationarity in the regression coefficients, thereby producing better predictive performance. This clearly indicates that our proposed model produces more stable cross-elasticity estimates.

Empirical results and discussions

The proposed model produces insights about cross-category price effects at various levels. First, the mean estimates $\hat{\mu}_j$ give the retailer a summary of what is happening at the chain level across time. Second, the heterogeneity factor loadings (λ) provide useful insights into how price elasticities co-vary across stores. Third, the heterogeneity loadings, combined with store factor scores, produce store-level estimates of cross-elasticities within and across categories. Finally, the nonstationary loadings (γ), combined with weekly factor stores, produce estimates for longitudinal trends in brand intercepts and elasticities.

⁷ The assumptions of independence and stationarity are necessary because the full-covariance formulation is not feasible.

⁵ Throughout the paper, we use lowercase to reference toothpaste, and uppercase to reference TOOTHBRUSH.

⁶ Since our main interest is cross-elasticities, a logarithmic functional specification is appropriate in our context. Nevertheless, we compared a log specification to a linear form using a non-nested P-E test (Davidson and MacKinnon 1981) as detailed in Greene (2003). The results for all 17 brands were conclusive, strongly rejecting the null of a linear specification.

Table 2
Model comparisons: estimation and prediction

Model	Estimation			Prediction	
	No. of parameters	Log-likelihood	BIC	RMSE	MAE
Aggregate SUR (a)	306	−44421	92167	0.7326	0.5511
Store-level SUR (b)	19278	− 14994	239490	0.8303	0.5826
Random-coefficients model (c)	612	−37535	81755	0.6819	0.4931
Random-coefficients model (d)	612	−57731	122150	0.8869	0.6845
Kalman-filter model (e)	612	−38512	83675	0.7999	0.6020
Factor-regression model	1530	−27240	71107	0.6569	0.4827

RMSE: root mean squared error, MAE: mean absolute error; Because the store-level model could not be fitted to 4 of the 66 stores, this model comparison is based only on 62 stores.

Note: Boldface type indicates the best model performance.

Chain-wide cross-brand and cross-category effects

Table 3a shows average chain-wide price-elasticities for each toothpaste brand on the sales of all brands in both categories. All, but one (*metadent*), of own-elasticities are negative and statistically significant as one would expect. As for cross-elasticities, we found five apparently counter-intuitive (i.e., negative) and statistically significant estimates out of a total of 72 (6.9 percent), which is lower than the 10 percent reported in a recent review of market share, sales and choice models in the literature (Sethuraman et al. 1999). Moreover, in a sales response model such as ours, negative cross-elasticities are not as implausible as in share or choice models, because of category volume effects. The highest within-category cross-elasticities we find are for *colgate* and *crest*, the dominant brands of toothpaste. A 1 percent price cut by *colgate* produces an average of 1.5 percent decrease in sales for *closeup* and *ultrabrite*, and an unexpected 1.5 percent increase in sales for the store brand (*Dominick's*). A similar pattern is seen for *crest*; a 1 percent price cut results in an average sales decrease of 1.7 percent and 1.5 percent for *close-up* and *ultrabrite*, respectively, and a 1.6 percent increase in the sales for the private label.

The patterns of cross-elasticities of the two leading national brands on the other national brands and private label are quite strong and consistent, despite the apparently counter-intuitive negative

cross-elasticities on the private label (i.e., an increase in sales for the private label when *crest* or *colgate* offer a discount). Our first conjecture for this odd complementarity effect between the leading brands and the store brand within the same product category was the possibility that the retailer schedules price promotions for its own brand when it was also promoting the leading brands, in an attempt to “free ride” on their promotion (for example, by placing the lower-priced store brand near the promoted leading brand). However, an analysis of prices did not show any strong positive correlation between the leading brands and the store brand. The complementarity might still be explained by proximity in shelf positioning, but unfortunately we do not have the data to confirm it.

Two strong cross-elasticities across categories are worthwhile to note in Table 3a: a 1 percent price cut by *colgate* in the toothpaste category produces 2.7 percent increase in *COLGATE* brush sales and 1.5 percent decrease in *ORALB* brush sales. Thus, we confirm, at the chain level, the similar types of within-category substitution and cross-category complementarity effects reported by Walters (1991) and by Mulhern and Leone (1991) for one store.

The average expected sales response to price changes in the toothbrush category are reported in Table 3b. As in the toothpaste category, most of own-elasticities are negative and statistically significant, with exceptions of *COLGATE* and *DOMINICK'S* (i.e., negative but not statistically significant). Overall, cross-elasticities

Table 3a
Average cross-elasticities for toothpaste brands (lowercase) on all others

SALES	PRICE CHANGES								
	aim	aquafresh	arm&ham	closeup	colgate	crest	dominic	metadent	ultrabrite
aim	-2.74	0.43	0.65	0.08	-0.24	-0.10	-0.07	1.01	0.26
aquafresh	0.19	-0.90	0.04	-0.25	0.90	0.45	-0.19	0.63	0.39
arm&ham	0.25	-0.27	-1.98	-0.09	-0.06	-0.29	-0.10	0.58	0.33
closeup	0.19	-0.37	-0.40	-1.39	1.48	1.67	-0.36	0.29	0.40
colgate	0.20	0.09	-0.05	0.13	-3.72	0.99	0.07	-0.05	0.04
crest	0.12	0.14	-0.10	0.10	0.74	-2.10	0.09	0.21	0.18
dominic	0.55	-0.47	-0.04	-0.13	-1.46	-1.58	-2.20	-0.29	-0.43
metadent	-0.31	0.24	0.10	0.43	0.08	0.81	0.18	-0.55	-0.09
ultrabrite	0.24	0.25	0.32	0.09	1.46	1.46	-0.80	-0.60	-3.38
AQUAFRESH	-0.06	0.79	0.10	-0.04	0.28	-0.15	-0.92	0.22	0.46
BUTLER	-0.27	0.02	-0.29	0.08	-0.72	0.03	0.39	0.09	-0.17
COLGATE	0.17	0.12	0.03	0.19	-2.73	0.75	-0.14	-0.19	0.20
CREST	0.02	0.03	0.01	-0.04	-0.31	-0.46	-0.38	0.17	0.19
DOMINIC	0.37	0.40	-0.24	0.10	0.12	-0.94	-0.01	0.31	0.07
ORALB	-0.12	0.33	0.03	0.21	1.54	0.83	-0.24	-0.10	0.16
PEPSODENT	-0.26	-0.18	0.09	-0.15	-0.12	-0.15	0.02	0.41	0.46
REACH	0.18	-0.37	-0.31	0.11	-0.49	-0.80	-0.12	0.44	0.15

Table 3b
Average cross-elasticities for TOOTHBRUSH brands on all others

SALES	PRICE CHANGES							
	AQUAFRESH	BUTLER	COLGATE	CREST	DOMINIC	ORALB	PEPSODENT	REACH
aim	0.23	0.08	0.09	0.55	-0.02	0.45	-0.28	0.13
aquafresh	-1.62	0.04	0.21	0.31	0.31	-0.42	0.21	-0.09
arm&ham	0.05	-0.05	0.66	0.29	-0.13	-0.65	0.33	-0.77
closeup	0.24	0.17	-0.57	-0.46	-0.22	0.00	-0.95	-0.26
colgate	-0.11	0.01	-0.18	0.22	-0.09	0.29	0.09	0.03
crest	0.11	0.01	0.06	-0.60	0.10	0.20	0.00	0.07
dominic	-0.04	-0.04	0.93	0.96	0.33	-0.24	0.18	-0.43
metadent	-0.02	0.11	0.08	0.07	-0.03	0.18	0.47	0.33
ultrabrite	0.26	0.12	0.03	-0.01	0.03	0.46	-0.32	-0.02
AQUAFRESH	-2.86	-0.07	0.14	-0.09	-0.02	-0.03	-0.14	0.21
BUTLER	0.25	-1.35	0.00	0.63	-0.01	0.11	0.17	0.37
COLGATE	0.07	0.11	-0.47	0.16	-0.10	0.30	-0.18	0.12
CREST	0.31	-0.09	0.26	-2.41	0.08	0.61	-0.01	0.12
DOMINIC	-0.08	0.12	0.46	0.37	-0.29	0.03	0.20	-0.05
ORALB	-0.52	-0.03	-0.40	-0.11	-0.14	-1.20	0.30	0.34
PEPSODENT	0.21	0.06	-0.14	0.29	0.05	0.66	-2.56	0.45
REACH	0.39	0.06	0.43	0.33	0.07	0.15	-0.58	-1.82

Note: Boldface type indicates that the corrected *t*-value of the elasticity is greater than 2.

within the toothbrush category are not as strong as those observed in the toothpaste category. As for cross-category effects, the only result worthy of note is the 1.6 percent expected increase in sales of *aquafresh* toothpaste in response to a 1 percent price cut by *AQUAFRESH* brush. These results, combined with those reported in Table 3a, show the same asymmetry in cross-category effects reported by Walters (1991). Similarly, a discount by *colgate* increases the sales of its “sister” brand in the toothbrush category, but not vice versa.

Although the cross-elasticities discussed above are consistent and useful, they might not fully reflect the managerially relevant impacts of a brand promotion, as they hide the large discrepancies in sales volume across brands. For example, in response to a 1 percent price cut by *CREST* brushes, the 1.6 percent increase in *aquafresh* sales might seem large compared to the increase of 0.18 percent in *crest* sales. However, when one takes into account that the average sales of *crest* is almost four times larger than that of *aquafresh*, it becomes clear that the retailer should look at sales response, in addition to elasticities. Another advantage of sales response is that they can be summed across the affected brands, summarizing the category impacts of the promoted brand.⁸ We estimate the incremental sales response to a 10 percent price discount as

$$\text{Sales response} = (\text{Average sales}) \times [1 - 0.9]^{\text{elasticity}}.$$

Table 4 shows the changes in sales expected in response to a 10 percent price discount in the toothpaste category. As one would expect, the leading brands in the category have substantial impact on total category sales. For example, a 10 percent discount on *colgate* would result in an increase of 315 units in its own sales, but due to brand switching would result in 214 incremental sales in the

product category. Similar results are observed for *crest*, where a 10 percent discount produces an increase in sales of 221 units for the brand, but only 123 units in incremental sales for the category. In contrast, the same price cut in the private label results in slightly higher incremental sales in the category (18 units) than in the brand’s own sales (16 units), suggesting that a promotion of the private label is less likely to draw sales from competing brands.

The small or negative incremental category sales for *aim*, *aquafresh*, *closeup*, *metadent*, and *ultrabrite* imply that a price discount by these brands is more effective in producing brand switching from competing brands than in attracting regular buyers of the brand. Thus, promoting these brands might be useful for the manufacturer, but not necessarily for the retailer. As for cross-category effects, the only results worth noting in Table 4 are that a 10 percent discount of *colgate* produces an increase of 9 units on *COLGATE* and a decrease of 6 units for *ORALB*, resulting in a net incremental growth of 5 units in the toothbrush category. The average sales changes in the toothbrush brands can be computed and interpreted in a similar way.

Store-level cross-brand and cross-category effects

As shown earlier, multiplying the factor loadings for a particular cross-elasticity (λ_j) by the factor scores for a given store (V_s) produces the deviation of the store *s* from the chain average (μ_j), resulting in store-level cross-elasticity “shrinkage” estimates. To illustrate this feature of the model, we report in Table 5 the category effects of a 10 percent discount by toothpaste brands for a sample of 10 individual stores, which can be compared to the chain-level results previously shown in Table 4. While *colgate* produces a substantially higher incremental effect in the toothpaste category than *crest* at the chain level (Table 4), Table 5 shows that at store #46 the incremental category effects for these two brands are fairly similar (199 and 186, respectively). Table 5 also shows that a discount by *Dominick’s* can have a positive effect in the toothpaste category at some stores (e.g. 1, 12, 23) and a negative effect on others (e.g., 15, 44, 46).

⁸ Category-level elasticities can be also obtained from the brand-level elasticities. For example, Hoch et al. (1995) defined a category-level elasticity as the category volume response produced by a uniform percentage change in all prices in the category. We look at the differential impact of a price discount by one specific brand.

Table 4
Incremental sales response to a 10 percent price discount by toothpaste brands (lowercase)

Sales	Average	10 Percent price discount								
		aim (\$0.24)	aquafresh (\$0.44)	arm & ham (\$0.54)	closeup (\$0.37)	colgate (\$0.37)	crest (\$0.39)	dominicme (\$0.26)	metadent (\$0.77)	ultrabrite (\$0.32)
aim	114	38	-5	-8	-1	3	1	1	-12	-3
aquafresh	248	-5	25	-1	7	-22	-12	5	-16	-10
arm & ham	169	-4	5	39	2	1	5	2	-10	-6
closeup	73	-1	3	3	12	-11	-12	3	-2	-3
colgate	656	-14	-6	3	-9	315	-65	-5	3	-2
crest	893	-11	-13	9	-9	-67	221	-8	-19	-17
dominic	60	-3	3	0	1	10	11	16	2	3
metadent	170	6	-4	-2	-8	-1	-14	-3	10	2
ultrabrite	94	-2	-2	-3	-1	-13	-14	8	6	40
Paste total	2479	2	4	41	-7	214	123	18	-38	4
AQUAFRESH	10	0	-1	0	0	0	0	1	0	0
BUTLER	15	0	0	0	0	1	0	-1	0	0
COLGATE	26	0	0	0	-1	9	-2	0	1	-1
CREST	18	0	0	0	0	1	1	1	0	0
DOMINIC	21	-1	-1	1	0	0	2	0	-1	0
ORALB	38	0	-1	0	-1	-6	-3	1	0	-1
PEPSODENT	11	0	0	0	0	0	0	0	0	-1
REACH	18	0	1	1	0	1	2	0	-1	0
BRUSH TOTAL	157	0	-2	1	-2	5	0	3	-2	-3

Store comparisons

One main feature of the factor-regression model is that the covariance of the random coefficients can be graphically represented in the latent space defined by the heterogeneity factors, showing how

price elasticities vary across stores. Moreover, stores can also be displayed in the same space, explaining how stores differ in their responses to price.

Fig. 1 displays the statistically significant factor loadings for brand intercepts on the heterogeneity factors. Brand intercepts

Table 5
Category effects of a 10 percent discount on toothpaste brands in different individual stores

Store	Category	Price changes								
		aim	aquafresh	arm & ham	closeup	colgate	crest	dominic	metadent	ultrabrite
1	paste	3	16	27	3	164	71	38	-20	1
	BRUSH	0	-1	1	-1	3	1	2	0	-2
12	paste	7	15	28	1	160	72	33	-32	1
	BRUSH	0	-1	1	-1	3	0	3	-1	-2
15	paste	-13	-34	84	-33	225	178	-32	-80	-20
	BRUSH	0	-4	2	-4	1	-8	2	-9	-5
23	paste	3	8	42	-1	279	113	27	-25	13
	BRUSH	-1	-3	1	-1	10	2	2	0	-2
37	paste	-17	-2	33	-2	344	89	17	-7	11
	BRUSH	-1	-3	1	-2	14	2	2	2	-1
44	paste	-9	-39	78	-38	228	177	-36	-81	-13
	BRUSH	0	-6	2	-5	-1	-13	2	-10	-6
46	paste	-19	-29	62	-34	199	186	-31	-66	-3
	BRUSH	0	-6	2	-4	1	-9	2	-8	-6
59	paste	90	17	7	47	210	101	20	23	39
	BRUSH	-1	0	1	-1	13	5	1	5	0
61	paste	7	0	35	-4	196	120	16	-26	20
	BRUSH	-1	-3	1	-2	5	2	2	-1	-2
65	paste	-39	-81	135	-64	315	296	-88	-123	-26
	BRUSH	-1	-9	2	-8	-2	-21	0	-17	-9

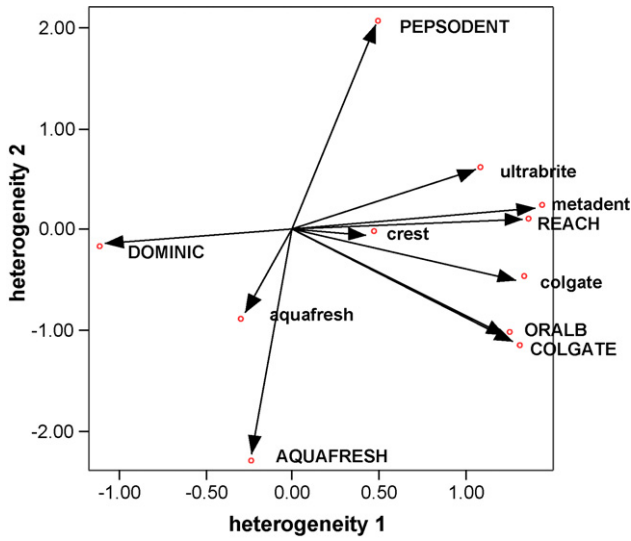


Fig. 1. Intercept.

represent the baseline sales for the brand after accounting for the effects of all (own and competitors) prices. Each vector in Fig. 1 points to the direction (in the heterogeneity factor space) where a store would have higher-than-average intercepts. Therefore, stores with factor scores located in the north side of Fig. 1 have higher-than-average baseline sales for PEPSODENT, the least expensive brand in the category. Stores with factor scores located in the south side of Fig. 1 have larger-than-average baseline sales for AQUAFRESH, the most expensive brand of toothbrushes. Stores located in the west side have higher-than-average baseline sales for DOMINICK'S, while those in the opposite direction have higher-than-average sales for national brands in the two categories.

Fig. 2 shows the statistically significant heterogeneity factor loadings for the impact of a leading national brand (crest and CREST) on brand sales in both categories. The directions of the vectors represent where the respective cross-elasticities are larger

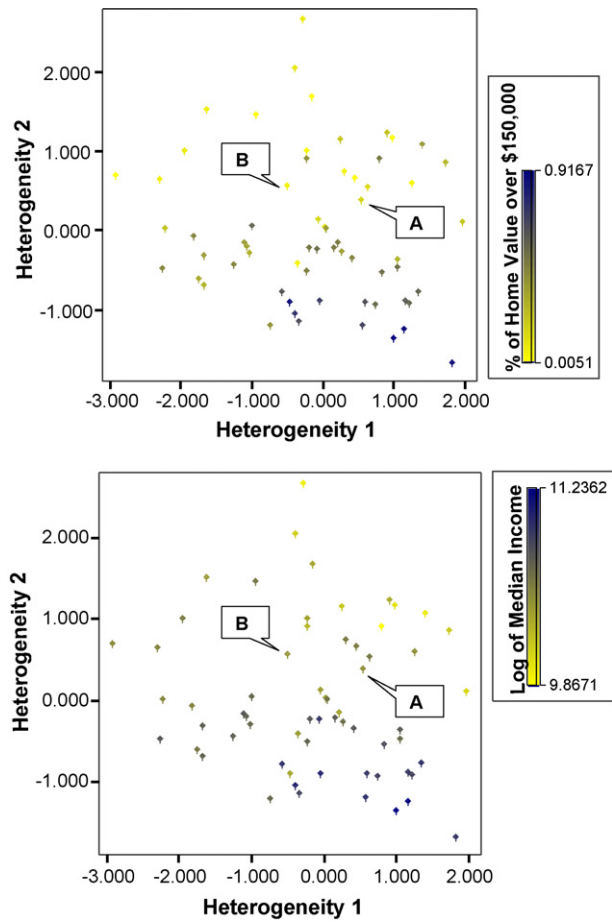


Fig. 3. Factor scores for each store by trade-area demographics.

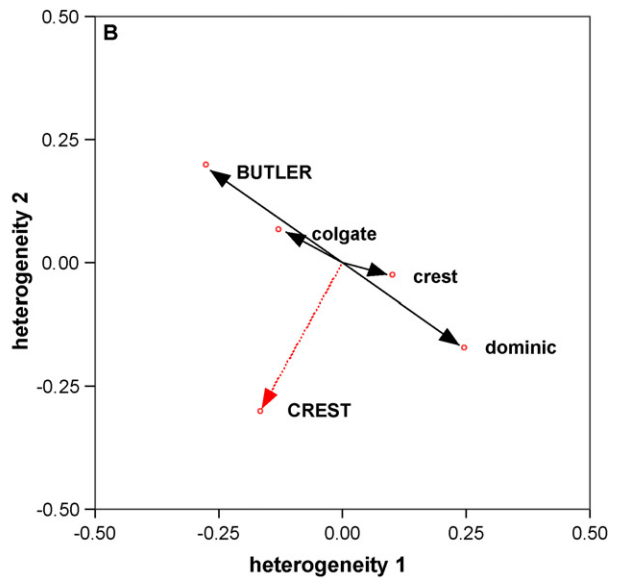
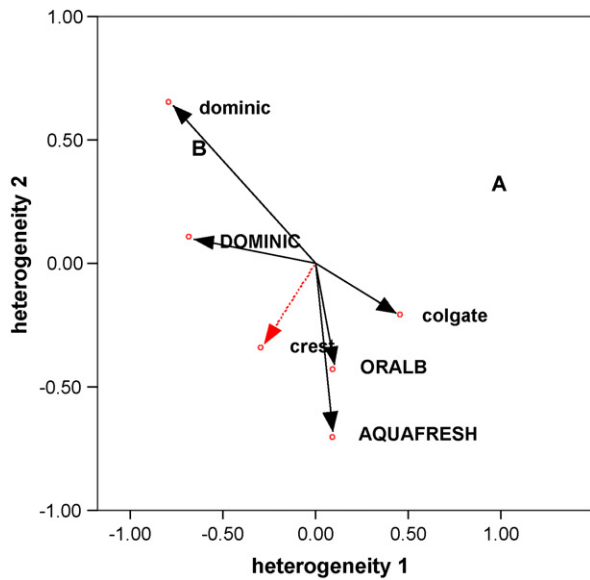


Fig. 2. Price-promotion effect of a leading brand of toothpaste (crest) and toothbrush (CREST) on both categories.

than average. However, one must take special care in interpreting the own-elasticities represented by traced vectors in Fig. 2. The traced vector points to the direction where own-elasticities are greater than average, but because own-elasticities are negative, that is the direction where own-elasticities are less negative (or weaker). For example, a store located in point A in Fig. 2a, away from the direction of the traced vector for *crest*, has customers who are more responsive to a price discount by *crest* than average. In contrast, the (solid) vectors for the cross-elasticities point to the direction where they are stronger than average. For example, the same store A shown in Fig. 2a also has higher than average cross-elasticities for the sales of *colgate* in response to a promotion by *crest*, suggesting that *colgate* is likely to lose more sales than average in store A due to a promotion by *crest*. Store B, on the other hand, shows a larger than average drop (or smaller than average increase) in sales of *dominic* in response to a price promotion by *crest*. Following these guidelines, one can conclude that stores located in the northeast of the heterogeneity factor space are more sensitive than average to the prices of *crest* and *CREST* (more negative own-elasticities

than average). In fact, a look at all own-elasticities (not shown here due to space constraints) leads to the same conclusion that stores with high scores on both factors (i.e., positioned in the northeast sector of Fig. 2) have customers who are more price sensitive than average.

Fig. 2a also shows that stores where the impact of a *crest* price promotion on *Dominick's* is higher than average also tend to show higher than average effect of *crest* on *DOMINICK'S*, suggesting that a *crest* promotion has cross-category effects on the private label. Fig. 2b shows that a *CREST* price promotion has a higher than average draw from *BUTLER* in stores located in the northwest of the heterogeneity factor space, such as store B.

Plotting the factor scores for each of the 66 stores in the same space as the toothpaste and toothbrush brands will allow us to identify the stores located in the more- and less-than-average price-sensitive areas of this latent space. Fig. 3 shows these plots, with each store denoted by different color shades, depending on the nature of its trade area. The conclusion from both panels of Fig. 3 is that stores located in the price sensitive region (NE) of the heterogeneity factor

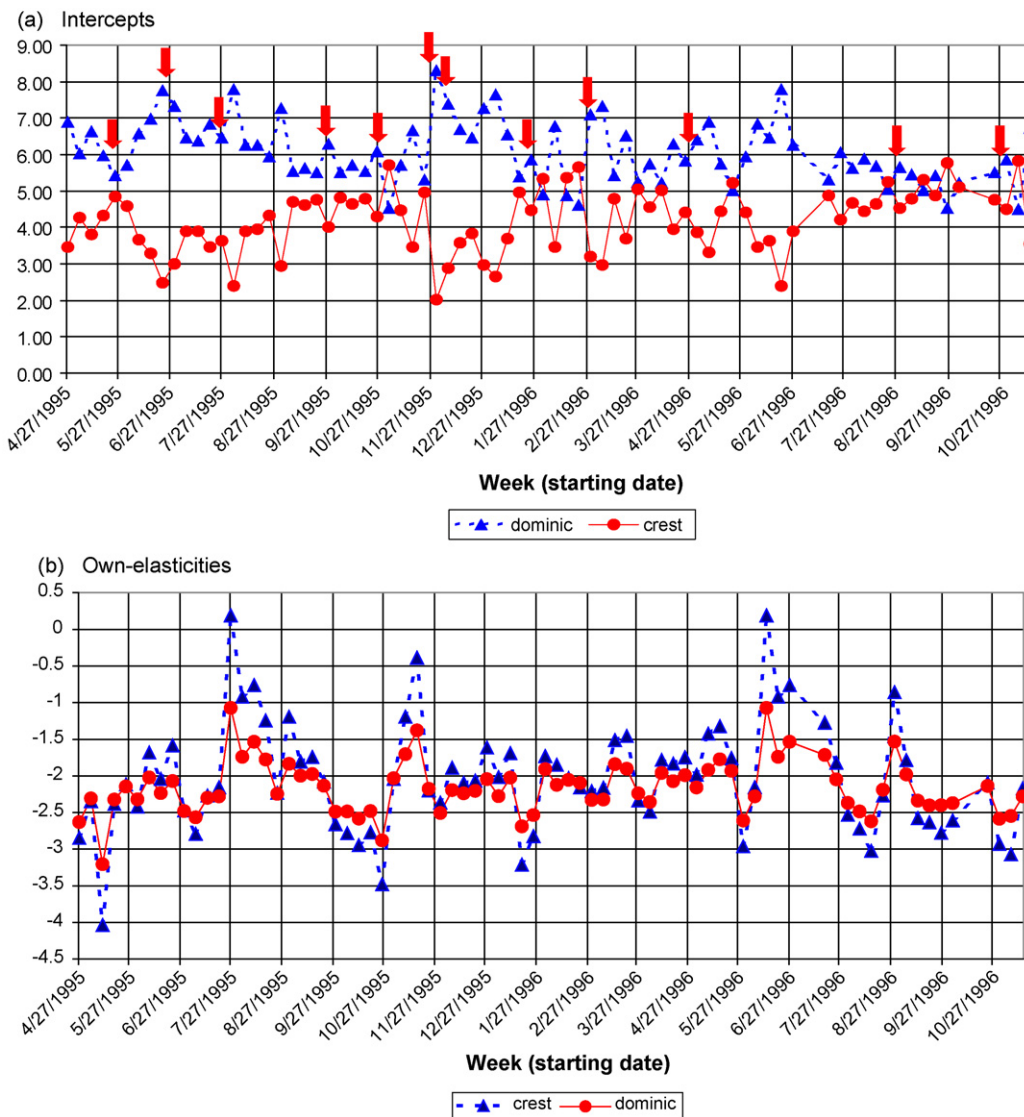


Fig. 4. Weekly trends for intercepts and own-elasticities for two toothbrush brands.

space tend to serve markets with lower median income and lower home values than the stores located in the less price-sensitive areas.

At this point, we must add an important caveat to the plots depicted in these figures: one must resist the temptation to interpret the vertical and horizontal axis in these plots. Factor models such as the one we propose here are known to be invariant to orthogonal rotation (Wedel and Kamakura 2001). In other words, any orthogonal rotation of the loadings (λ , γ) and factor scores (V , W) would produce identical fit to the data, and therefore are as suitable as any other orthogonal rotation. As a result, interpretation of the underlying factors or dimensions is highly subjective. On the other hand, the relative position of brands and stores in these plots remains constant regardless of orthogonal rotations. That is, they can be interpreted regardless of the arbitrary rotation, without any loss of generality.

Longitudinal analysis

Another useful feature of our proposed model is that it also allows for the regression coefficients to change over time. Since the best fitting formulation of our model utilizes two nonstationary factors (see Table 1), the model allows the regression coefficients to follow different longitudinal patterns, depending on their loadings on each of the two factors. They may also provide some insights to the chain manager regarding the general longitudinal trends. As an illustration, we show in Fig. 4 the longitudinal trends for brand intercepts and own-elasticities of the leading brand (*crest*) and private label (*Dominick's*) in the toothpaste category. One can see that the intercepts of the two brands, which can be interpreted as baseline sales after accounting for all prices and store differences, have trends that are mirror images of each other (Fig. 4a), while their own-elasticities follow essentially the same trend. Unfortunately, we do not have any managerial insights that would explain the shifts in trend for the intercepts and elasticities. These trends do not seem to be related to holidays or to any seasonality, which is expected since the two product categories are not likely to be affected by these time-related factors. We conjecture that our dataset is not long enough to catch any structural changes. Nevertheless, Fig. 4 serves to highlight the potential usefulness of the proposed model in detecting longitudinal changes in brand “attractiveness” (intercepts) and in consumers’ sensitivity to price, or to test for the possible impact of observed market disruptions such as the repositioning of an existing brand, new brand introduction, new retail chain, and so on. In addition, it is important to note that these two factors allow the model to produce estimates of cross-elasticities devoid of nonstationarity biases.

Conclusions and directions for future research

The main purpose of this study is to present a relatively simple, feasible and easy-to-implement approach for chain-wide, store-level cross-category analysis. This analysis is intended to help retail managers make both chain-wide and store-specific decisions. Our model produces more precise average estimates of cross-category elasticities for the chain, while accounting for unobserved heterogeneity across stores and nonstationarity over time.

From a substantive point of view, we confirm some of the results found in previous studies. Unlike these previous results obtained for individual stores, we generalize the

conclusions by demonstrating how a retail chain can gain similar insights regarding the cross-category effect of its price-promotions across all stores. In doing so, our proposed model provides more precise and robust “global” chain-level estimates, while also producing “local” store-level estimates, taking advantage of all the information available to the chain. This distinction is critical, because aggregate estimates suffer from pooling biases and estimates obtained from each individual store are unreliable due to the limited degrees-of-freedom. By taking advantage of all the information to obtain the individual estimates, our approach leads to more stable estimates, as we demonstrated empirically through predictive tests. Moreover, the model also provides additional insights about how the cross-elasticities vary across stores (through the factor loadings), and how the stores differ in the price sensitivity across their market areas (through the factor scores).

One main limitation of our approach (and of previous attempts to estimate brand-level cross-elasticities across categories using store-level data) is that we only consider immediate effects, observed within the same week of the sales promotions, thereby ignoring any possible residual effects of these promotions. This is particularly critical given recent evidence of postpromotion cancellation effects reported by Nijs et al. (2001) based on an extensive study of 560 product categories using aggregate (national) weekly data at the category level. Kopalle et al. (1999) also discuss several sources of dynamics in baseline sales and price sensitivity. Therefore, a vector autoregressive (Nijs et al. 2001) or varying-parameter (cf. Kopalle et al. 1999) formulation might be needed, beyond controlling for the nonstationarity in parameters. To investigate this possibility, we examined the residuals of each of the 17 regressions for each of the 66 stores in our sample, but found no consistent evidence of serial correlation in the residuals. Thus, we concluded that an autoregressive formulation was not needed in analyzing our data, after accounting for nonstationarity.

Clearly, price is not the only marketing stimuli. Unfortunately, however, we did not have access to data on other types of marketing stimuli, such as feature advertising or shelf location. Given its parsimonious formulation, the proposed factor-regression model would allow us to estimate average cross-elasticities on these stimuli as well. In fact, the benefits we found due to parsimony would be even more accentuated as the number of cross-elasticities to be estimated increases. More importantly, the factor structure would also provide valuable insights into the relationship between responses to feature advertising and price, for example. These additional data would also allow us to further examine the effects of brand and store characteristics on responses to marketing stimuli (cf. Karande and Kumar 1995). We believe that this stream of research will be valuable, and hope that our framework can facilitate such attempts.

While we limited our analyses to two related product categories for illustration purposes, the model is easily applicable to multiple categories. Although the number of parameters will increase considerably, our factor-regression model will

still be feasible, while competing approaches such as random-coefficients regression will not. For instance, the model could be useful for studying the impact of a store brand across all categories, by covering a broad range of product categories, but limiting the analyses to groups of brands (e.g., private label vs. national brands). By analyzing multiple categories, one could examine any potential differences in price sensitivity across product categories (e.g., functional vs. hedonic, Wakefield and Inman 2003).

The basic factor-regression formulation can also be easily extended to other types of response models involving multivariate dependent variables. One such extension could be a multivariate Tobit model for basket analysis, in which a Tobit-regression model is specified for the (possibly truncated) quantity observed in each product category as a function of price indices for all categories. The category-level Tobit-regression models would then be “linked” across categories using a similar factor structure as the one we specify in (2), leading to a multivariate Tobit factor-regression model for market basket analysis.

Finally, we warn readers against drawing any generalization based on the empirical results presented in this study, because they are limited to two product categories across the multiple stores of a single retail chain. As in any empirical study such as ours (and others in the marketing literature), such generalizations would be warranted only after consistent replications across multiple product categories and markets are obtained.⁹ We also note that we only had access to data on price promotions, and could not consider other drivers of sales response such as feature advertising, display and coupons. This omission could potentially bias our elasticity estimates and these variables could be empirically explored in future research.

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⁹ We thank an anonymous reviewer and the AE for reminding us about this important caveat.

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